EX NAVODAYAN FOUNDATION
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Full Syllabus

JEE-Main

Paper-1

Test Date: **M.M: 300**

TEST INSTRUCTIONS

1. The test is of **3 hours** duration.
2. The test booklet consists of **75 questions**.
3. The maximum marks are **300**.
4. All questions are compulsory.
5. There are three parts in the questions paper consisting of Physics, Chemistry and Mathematics having **25 questions in each part**.

Each Parts Contains –

- 20 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct. All questions are carrying **+4 marks** for right answer and **-1 mark** for wrong answer.
- 05 questions with answer as **numerical value** all questions are carrying **+4 marks** for right answer and **–1 marks** for wrong answers.

Name of the Candidate (in Capital Letter): _____

Registration No. _____

Invigilator Signature

_____

Physics

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. The temperature of equal masses of three different liquids A, B and C are 12°C, 19°C and 28°C respectively. The temperature when A and B are mixed is 16°C and when B and C are mixed is 23°C. The temperature when A and C are mixed is _____ °C.

   (a) 20.2°C (b) 15.2°C (c) 10.2°C (d) 25.2°C

2. Consider the following equation of Bernoulli's theorem, $P+\frac{1}{2}\rho v^2+\rho gh=K(\text{Cont})$. The dimensions of K/P are same as that of which of the following

   (a) Thrust (b) Presence (c) Angle (d) Viscosity

3. A solid cylinder is rolling down on an inclined plane of angle θ. The coefficient of static friction between the plane and cylinder is μ_s. The condition for the cylinder not to slip is

   (a) $\tan\theta \geq 3\mu_s$ (b) $\tan\theta > 3\mu_s$ (c) $\tan\theta \leq 3\mu_s$ (d) $\tan\theta < 3\mu_s$

4. The potential energy of a 1 kg particle free to more along the x-axis is given by $v(x)=\left(\frac{x^4}{4}-\frac{x^2}{2}\right)J$.

   The total mechanical energy of particle is 2J. Then the maximum speed (in m/s) is

   (a) 2.121 m/s (b) 3.421 m/s (c) 4.321 m/s (d) 5.321 m/s

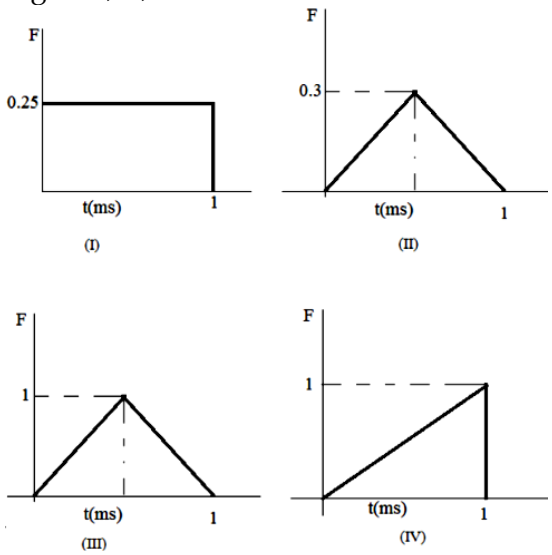
5. Consider a drop of rain water having mass 1g falling from a height of 1 km. It hits the ground with a speed of 50 m/s. Take g = 10 m/s². The work done by the resistance force of air is _____ Joule.

   (a) –8.75 (b) –8.0 (c) –10.75 (d) –10.0

6. A coin of mass 10 g rolls along a horizontal table with a velocity of 6 cm/s. Its total kinetic energy is

   (a) 9 μJ (b) 18 μJ (c) 27 μJ (d) 36 μJ

7. Figure I, II, III & IV direct variation of force with time



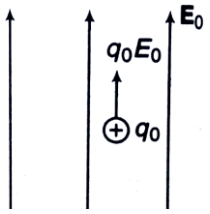
The impulse is highest in case of situation depicted figure

(a) I & II (b) III & I (c) III & IV (d) IV only

8. The linear density of a rod of length 3 m varies as $\lambda = 2t$, then the position of the center of mass of the rod is

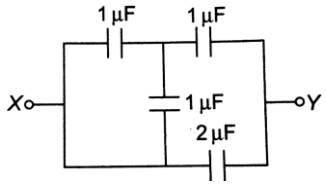
(a) $\frac{7}{3}$m (b) $\frac{12}{7}$m (c) $\frac{10}{7}$m (d) $\frac{9}{7}$m

9. A uniform electric field E_0 is directed along positive y-direction. Find the change in electric potential energy of a positive test charge q_0 when it is displaced in this field from $y_i = a$ to $y_f = 2a$ along the y-axis.



(a) q_0E_0a (b) $-q_0E_0a$ (c) $\frac{q_0E_0a}{2}$ (d) $-\frac{q_0E_0a}{2}$

10. The equivalent capacitance between x & y is



(a) $\frac{5}{6}\mu F$ (b) $\frac{7}{6}\mu F$ (c) $\frac{8}{3}\mu F$ (d) $1\mu F$

11. When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{7}$ (d) $\frac{5}{7}$

12. A ball is thrown vertically downward from a height of 20 m with a initial velocity v_0. It collides with the ground looses 50% of its energy in collision and rebounds to the same height. The initial velocity v_0 is _____ (in m/s)

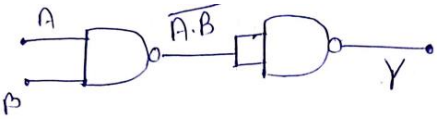
(a) 20 (b) 40 (c) 10 (d) 60

13. A conducting loop has an area of $A = 3 \times 10^{-4}$ m^2 and resistance 0.05 Ω. A magnetic field of 70.0 μT is strong perpendicular to the plane of the loop. If the area charges at rate, $\frac{dA}{dt} = -5 \times 10^{-6}$ m^2/s, this magnitude of current induce in the loop is

(a) 70×10^{-10} A (b) 3.5×10^{-10} A (c) 70×10^{-12} A (d) 3.5×10^{-12} A

14. The wavelength of light for the energetic proton emitted in the lyman series of hydrogen atom spectrum lies in the range of

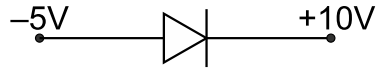
(a) ultraviolet (b) Infrared range (c) visible range (d) x-rays range

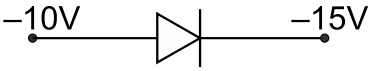
15. 

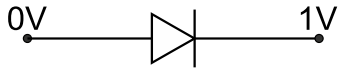
(a) AND ($\alpha = 1, \beta = 1, \gamma = 1$) (b) NOT ($\alpha = 1, \beta = 1, \gamma = 1$)

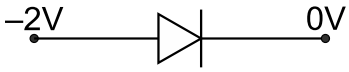
(c) OR ($\alpha = 1, \beta = 1, \gamma = 0$) (d) XOR ($\alpha = 0, \beta = 0, \gamma = 0$)

16. In which can junction diodes is not reverse biased?

(a) 

(b) 

(c) 

(d) 

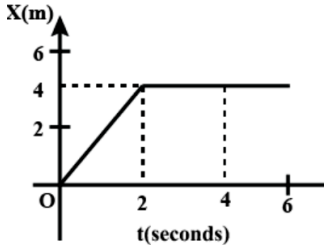
17. $X = 3YZ^2$ Find dimension of Y in (MKSA) system, if X and Z are the dimension of capacity and magnetic field respectively

(a) $M^{-3}L^{-2}T^{-4}A^{-1}$ (b) ML^{-2} (c) $M^3L^2T^4A^4$ (d) $M^{-3}L^{-2}T^8A^4$

18. A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 sec. is

(a) 10 unit (b) $7\sqrt{2}$ unit (c) 7 unit (d) 85 unit

19. In the figure given below, the position tune graph of a particle of mass 0.1 kg is shown. The impulse at t = 2 sec. is



(a) 0.2 kg ms^{-1} (b) –0.2 kg ms^{-1} (c) 0.1 kg ms^{-1} (d) –0.4 kg ms^{-1}

20. From a balloon rising vertically upward 5 m/s a stone is thrown up at 10 m/s relative to balloon the velocity with respect to ground after 2 sec. is (assume $g = 10\ m/s^2$)

(a) 0 m/s (b) 20 m/s (c) 10 m/s (d) 5 m/s

(Integer Type Questions)

This Section contains **05 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

21. Two particles executes SHM of same amplitude and frequency along the same straight line. They pass one another when going in opposite directions and each time their displacement is half of their amplitude. The phase difference between them is _____ (in degree)

22. The temperature of the mixture of one mole of helium and one mole of hydrogen is increased from 0°C to 100°C at constant pressure. The amount of heat delivered will be _____ (in calorie)

23. The maximum horizontal range of projectile is 400 m. The maximum value of height attained by it will be _____ (in meter)

24. The work function of a substance is 4 eV. The longest wavelength of light that can cause photo electron emission from this substance is nearly _____ (in nm)

25. At what temperature is the root mean square velocity of gaseous hydrogen molecules equal to that of oxygen at 47°C is _____ (in kelvin)

Chemistry

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. At 25°C and 1 atm pressure the enthalpy of combustion of Benzene (ℓ) and acetylene (g) are –3268 kj/mol and –1300 kj/mol respectively. The change in enthalpy of reaction

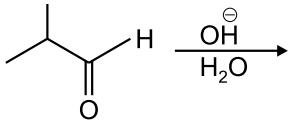
$$3C_2H_2(g) \rightarrow C_6H_6(\ell) \text{ is}$$

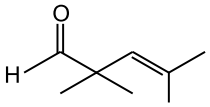
(a) +324 kj (b) +632 kj (c) –632 kj (d) –732 kj

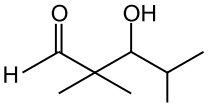
2. Which of the following options of species have identical shape

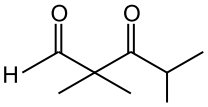
(a) $BeCl_2$, XeF_2, CO_2 (b) PF_5, IF_5, IF_7 (c) BF_3, NH_3, PCl_3 (d) CF_4, SF_4, XeF_4

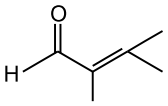
3. What is the major product of the following reaction



(a) 

(b) 

(c) 

(d) 

4. What is the PH of the solution. If the cell potential for the cell $Pt/H_2(g)/H^+(aq)/Cu^{2+}(0.01\ M)/Cu(s)$ is 0.576V at 298 K. Given $E^o_{Cu^{2+}/Cu} = 0.34$ V

(a) 4 (b) 9 (c) 5 (d) 2

5. Given below are two statements one labelled as Assertion (A) and the other as Reason (R):

Assertion (A): Thin layer chromatography is an adsorption chromatography.

Reason (R): A thin layer of silica gel is spread over a glass plates suitable size in thin layer chromatography which act as an adsorbent. In the light of the above statement choose the correct answer from the option given below:

(a) Both A and R are True and R is correct explanation of A.

(b) Both A and R one true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

6. Find the work done when 2 moles of hydrogen expanded isothermally from 15 to 50 litres against a constant pressure of 1 atm at 25°C.

(a) 847 cals (b) 847 kcal (c) 84.7 cals (d) 84.7 kcal

7. In which of the following species oxidation number per atom of the underlined elements is are equal to +1.

(a) $\underline{S}_2O_3^{2-}, \underline{P}_3O_9^{3-}$ (b) $\underline{P}_3O_9^{3-}, \underline{N}_2O$ (c) $H_3\underline{P}O_2$, $\underline{Fe}_2O_3$ (d) $\underline{N}_2O$, $H_3\underline{P}O_2$

8. The decomposition of N_2O_4 to NO_2 was carried out in chloroform at 280K. At equilibrium 0.2 mol of N_2O_4 and 2×10^{-3} mol of NO_2 were present in 2L of solution. The equilibrium constant for the reaction $N_2O_4 \rightleftharpoons 2NO_2$ is

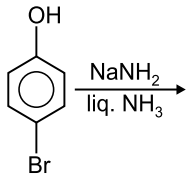
(a) 0.01×10^{-4} (b) 2×10^{-3} (c) 2×10^{-5} (d) 1×10^{-5}

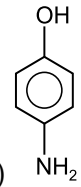
9. Boric acid is:

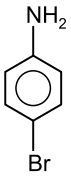
(a) Monobasic and weal Lewis acid (b) Tribasic and strong Lewis acid

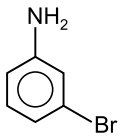
(c) Monobasic and weak Bronsted acid (d) Tribasic and weak Bronsted acid

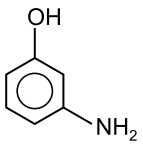
10. The major product of the following reaction is:



(a) 

(b) 

(c) 

(d) 

11. The major product of the following reaction is $CH_3-\underset{\displaystyle Cl}{\underset{|}{C}}H-CH_3 \xrightarrow[\text{(ii) HBr peroxide}]{\text{(i) Alc KOH}} \xrightarrow[\text{(iii) aq. KOH}]{}$

(a) $CH_3-\underset{\displaystyle OH}{\underset{|}{C}}H-CH_3$ (b) $CH_3-CH_2-CH_2-OH$

(c) $CH_3-CH=CH_2$ (d) $CH_3-\underset{\displaystyle Br}{\underset{|}{C}}H-CH_3$

12. In SN^1 reaction the correct order of reactivity for the following compounds CH_3Cl, CH_3-CH_2-Cl $(CH_3)_2CHCl$ and $(CH_3)_3CCl$ is

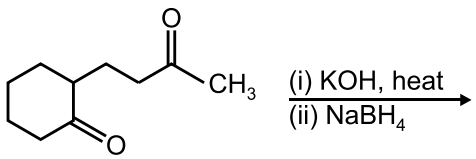
(a) $(CH_3)_3CCl > (CH_3)_2CHCl > CH_3Cl > CH_3\text{–}CH_2\text{–}Cl$

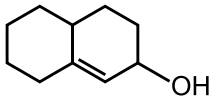
(b) $(CH_3)_3CCl > (CH_3)_2CHCl > CH_3\text{–}CH_2\text{–}Cl > CH_3\text{–}Cl$

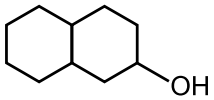
(c) $CH_3\text{–}Cl < (CH_3)_2CHCl > CH_3CH_2Cl > (CH_3)_3CCl$

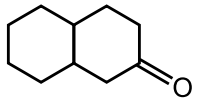
(d) $CH_3Cl > CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3CCl$

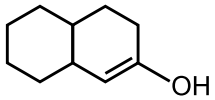
13. The major product of the following reaction is



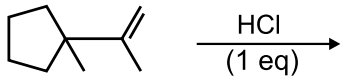
(a) 

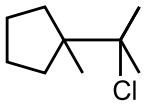
(b) 

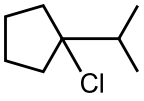
(c) 

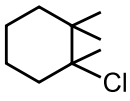
(d) 

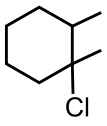
14. Major product of the reaction is



(a) 

(b) 

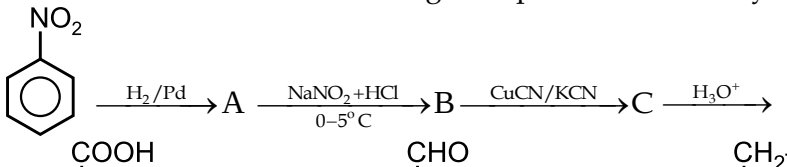
(c) 

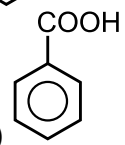
(d) 

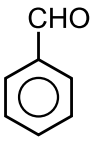
15. Among the following statement on the nitration of aromatic compounds, the false one is
(a) The rate of nitration of Benzene is almost the same as that of hexadeciterobenzene
(b) The rate of nitration of Toluene is greater than that of Benzene
(c) The rate of nitration of Benzene is greater than that of hexadeuterobenzene
(d) Nitration is an electrophilic substitution reaction

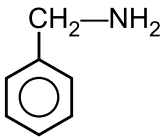
16. To prepare 3-ethylpentane-3-ol the reactants needed are:
(a) $CH_3CH_2MgBr + CH_3ClCH_2–CH_3$ (b) $CH_3MgBr + CH_3CH_2–CH_2COCH_2–CH_3$
(c) $CH_3–CH_2MgBr + CH_3CH_2COCH_2–CH_3$ (d) $CH_3CH_2CH_2MgBr + CH_3COCH_2CH_3$

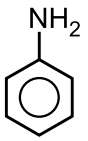
17. In a set of reaction nitrobenzene gave a product D. Identify the product D



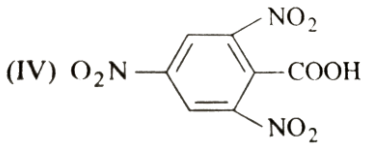
(a) 

(b) 

(c) 

(d) 

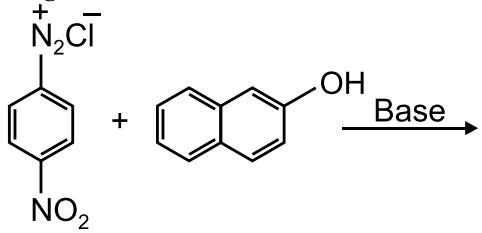
18. Give the order of decarboxylation of the following acid:
CH_3COOH ; $CH_2 = CH–CH_2–COOH$;
I II

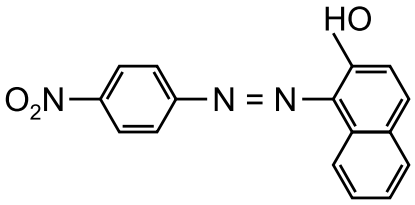
$CH_2(COOH)_2$; (IV) 

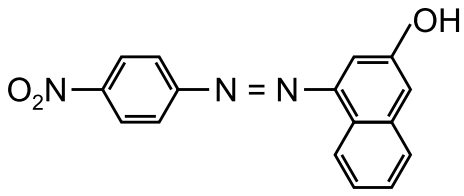
III

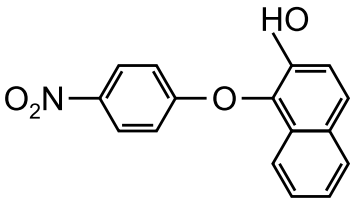
(a) I > II > III > IV (b) III > IV > II > I (c) IV > III > II > I (d) I > III > II > IV

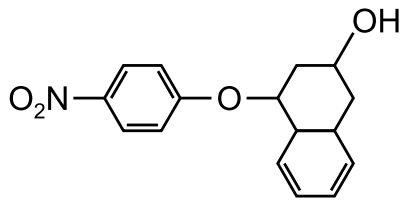
19. The major product of the following reaction is:



(a) 

(b) 

(c) 

(d) 

20. The number of asymmetric carbon atom in the glucose molecule in open and cyclic form is

(a) Four, Five (b) Four, Four (c) Five, Four (d) Five, Six

(Integer Type Questions)

This Section contains **05 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

21. The specific rate constant of the decomposition of N_2O_5 is 0.008 min^{-1}. The volume of O_2 collected after 20 minutes is 16 ml. The volume that would be collected at the end of reaction NO_2 formed is dissolved in CCl_4 _____ ml.

22. The e.m.f. of cell $Zn|ZnSO_4||CuSO_4|Cu$ at 25°C is 0.03 V and the temperature coefficient of e.m.f. is 1.4×10^{-4} V per degree. The heat of reaction for the change taking place inside the cell is _____ kJ/mole.

23. The reaction between X and Y is first order with respect to X and zero order with respect to Y.

Experiment	[x]/mol L^{-1}	[Y]/mol L^{-1}	Initial rate/mol L^{-1} min^{-1}
I	0.1	0.1	2×10^{-3}
II	L	0.2	4×10^{-3}
III	0.4	0.4	$M \times 10^{-3}$
IV	0.1	0.2	2×10^{-3}

Examine the data of table and calculate ratio of numerical values of M and L. (Nearest Integer)

24. 2.4 g coal is burnt in a bomb calorimeter in excess of oxygen at 298 K and 1 atm pressure. The temperature of the calorimeter rises from 298 K to 300 K. The enthalpy change during the combustion of coal is –x kJ mol^{-1}. The value of x is _____ (Nearest Integer)

(Given : Heat capacity of bomb calorimeter is 20.0 kJ K^{-1}. Assume coal to be pure carbon)

25. If weight of the non-volatile solute urea ($NH_2-CO-NH_2$) to be dissolved in 100g of water, in order to decrease the vapour pressure of water by 25% then the weight of the solute will be ____ g.

Mathematics

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. If $f(x) = |x - 1| \cdot ([x] - [-x])$, then (where [.] represent greater integer function)

   (a) f(x) is continuous and differentiable at x = 1

   (b) f(x) is discontinuous at x = 1

   (c) f(x) is continuous at x = 2

   (d) f(x) is continuous but non differentiable at x = 1

2. A fair dice is thrown three times. The probability that the product of three outcomes is a prime number is

   (a) $\frac{1}{24}$ (b) $\frac{1}{36}$ (c) $\frac{1}{32}$ (d) $\frac{1}{8}$

3. If P(1, 0), Q(–1, 0) and R(2, 0) are three given points then the locus of S satisfying the equation $SQ^2 + SR^2 = 2SP^2$

   (a) a straight time parallel to x-axis (b) circle through origin

   (c) circle with center at the origin (d) a straight line parallel the y-axis

4. Suppose a solution of the differential equation $(xy^3 + x^2y^7)\frac{dy}{dx} = 1$ satisfies the initial conditions $y\left(\frac{1}{4}\right) = 1$. Then the value of $\frac{dy}{dx}$ when y = –1 is

   (a) $-\frac{3}{20}$ (b) $-\frac{20}{3}$ (c) $-\frac{5}{16}$ (d) $-\frac{16}{5}$

5. Let f be a function satisfying the functional equation $f(x) + 2f + \left(\frac{2x+1}{x-2}\right) = 3x,\ x \neq 2$. Then the value of $\frac{f(3)}{f(7)}$ is

   (a) 9 (b) 10 (c) –11 (d) 12

6. Suppose y = f(x) and y = g(x) are two functions whose graphs intersect at the three points (0, 4), (2, 2) and (4, 0) with f(x) > g(x) for 0 < x < 2 and f(x) < g(x) for 2 < x < 4. If $\int_0^4 [f(x) - g(x)]dx = 10$ and $\int_2^4 [g(x) - f(x)]\,dx = 5$, then the area between two curves for 0 < x < 2 is

   (a) 5 (b) 10 (c) 15 (d) 20

7. A be a square matrix of order 2 with $|A| \neq 0$ such that $|A + |A|adj(A)| = 0$, where adj(A) is a adjoint of matrix A, then the value of $|A - |A|adj(A)|$ is

   (a) 1 (b) 2 (c) 3 (d) 4

8. The total number of three-digit numbers, divisible by 3, which can be formed using the digits 1, 3, 5, 8 if repetition of digits is allowed is

(a) 22 (b) 18 (c) 20 (d) 21

9. $\int \frac{x + x^{2/3} + 2 \cdot x^{1/6}}{x(1 + x^{1/3})} dx$ equals

(a) $\frac{3x^{3/2}}{2} + \tan^{-1}\left(x^{1/6}\right) + C$ (b) $\frac{3}{2}x^{3/2} + 12\tan^{-1}\left(x^{1/6}\right) + C$

(c) $\frac{3}{2}x^{2/3} + 6\tan^{-1}\left(x^{1/6}\right) + C$ (d) None of these

10. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$ is

(a) 4 (b) 120 (c) 210 (d) 310

11. Let two non-collinear vectors $\vec{a}$ and $\vec{b}$ inclined at an angle $\frac{2\pi}{3}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = 2$. If a point P moves so that at any time t its position vector $\overrightarrow{OP}$ (where O is the origin) is given as $\overrightarrow{OP} = \left(t + \frac{1}{t}\right)\vec{a} + \left(t - \frac{1}{t}\right)\vec{b}$ then least distance of P from the origin is

(a) $\sqrt{2\sqrt{133} - 10}$ (b) $\sqrt{2\sqrt{133} + 10}$ (c) $\sqrt{5 + \sqrt{133}}$ (d) None of these

12. Let $H_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$ then the sum of n terms of the series $\frac{1^2}{1^3} + \frac{1^2 + 2^2}{1^3 + 2^3} + \frac{1^2 + 2^2 + 3^2}{1^3 + 2^3 + 3^3} + ...$ is

(a) $\frac{4}{3}Hn - 1$ (b) $\frac{4}{3}Hn + \frac{1}{n}$ (c) $\frac{4}{3}Hn$ (d) $\frac{4}{3}Hn - \frac{2}{3}\left(\frac{n}{n+1}\right)$

13. Let $x_1, x_2, \ldots x_n$ be n observations. Let $w_i = lx_i + k$ for i = 1, 2, …, n where *l* and k are constants. If the mean of x_i's is 48 and their standard deviation is 12, the mean of w_i's is 55 and standard deviation of w_i's is 15, the values of *l* and k should be

(a) $l = 1.25, k = -5$ (b) $l = -1.25, k = 5$ (c) $l = 2.5, k = -5$ (d) $l = 2.5, k = 5$

14. Let $d \neq 0$ be in R. If α and β are the roots of the equation $x^2 - x + 2d = 0$ and α and γ are roots of the equation $3x^2 - 10x + 27d = 0$, then $\frac{\beta\gamma}{d}$ is equal to

(a) 36 (b) 9 (c) 27 (d) 18

15. Let $S = \left\{z = x + iy : \frac{2z - 3i}{4z + 2i} \text{ is a real number}\right\}$. Then, which of the following is not correct?

(a) $(x, y) = \left(0, -\frac{1}{2}\right)$ (b) $y + x^2 + y^2 \neq -\frac{1}{4}$

(c) $x = 0$ (d) $y \in \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

16. The two circles $x^2 + y^2 - 5 = 0$ and $x^2 + y^2 - 2x - 4y - 15 = 0$

(a) Touch each other externally (b) Touch each other internally

(c) Cut each other orthogonally (d) Do not intersect

17. Number of integral value of λ for which $\lim\limits_{x\to 1} \sec^{-1}\left(\frac{\lambda^2}{\ln x} - \frac{\lambda^2}{x-1}\right)$ does not exist is

(a) 1 (b) 2 (c) 3 (d) 4

18. The line through the intersection of $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ and also through the point (2, 1, –2) meets xy-plane at

(a) (–4, –3, 0) (b) (–4, 2, 0) (c) (2, –3, 0) (d) (–2, 3, 0)

19. Let f be a continuous and differentiable function in (x_1, x_2). If $f(x)\cdot f'(x) \geq x\sqrt{1-(f(x))^4}$ and $\lim\limits_{x\to x_1^+}(f(x))^2 = 1$ and $\lim\limits_{x\to x_2^+}(f(x))^2 = \frac{1}{2}$, then minimum value of $x_1^2 - x_2^2$ is

(a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{3}$ (d) None of these

20. Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = 2^{j-i}\ \forall$ i, j = 1, 2, 3. Then the matrix $A^2 + A^3 +$ …. $+ A^{10}$ is equal to

(a) $\left(\frac{3^{10}-3}{2}\right)A$ (b) $\left(\frac{3^{10}-1}{2}\right)A$ (c) $\left(\frac{3^{10}+1}{2}\right)A$ (d) $\left(\frac{3^{10}+3}{2}\right)A$

(Integer Type Questions)

This Section contains **05 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

21. If $L = \lim\limits_{x\to 0}\left(\frac{1}{\ln(1+x)} - \frac{1}{\ln(x+\sqrt{1+x^2})}\right)$ then the value of 18 L is

22. Let f be a function defined on the interval $[0, 2\pi]$ such that $\int_0^x (f'(t) - \sin 2t)dt = \int_x^0 f(t)\tan t\ dt$ and f(0) = 1. Then the maximum value of 8 f(x) is

23. Line L_1 is parallel to vector $\vec{\alpha} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ and passes through a point A(7, 6, 2). Line L_2 is parallel to a vector $\vec{B} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through a point B(5, 3, 4). Now a line L_3 parallel to a vector $\vec{r} = 2\hat{i} - 2\hat{j} - \hat{k}$ intersects the lines L_1 and L_2 at point C and D respectively, then $|\overrightarrow{CD}|$ = ?

24. Two circles in the first quadrant of radii r_1 and r_2 touch the co-ordinate axes. Each of them cuts off an intercept of 2 units with the line x + y = 2. Then, $r_1^2 + r_2^2 - r_1 r_2$ is equal to

25. If P(h, k) be a point on the parabola $x = 4y^2$, which in nearest to the point Q(0, 33), then the distance of P from the directirx of the parabola $y^2 = 4(x + y)$ is equal to

Answer – Key

Physics

1.	A
2.	C
3.	C
4.	A
5.	A
6.	C
7.	C
8.	B
9.	B
10.	C
11.	D
12.	A
13.	A
14.	A
15.	A
16.	B
17.	D
18.	B
19.	B
20.	D
21.	120
22.	1200
23.	100
24.	310
25.	20

Chemistry

1.	c
2.	a
3.	b
4.	c
5.	a
6.	a
7.	d
8.	d
9.	a
10.	d
11.	b
12.	b
13.	a
14.	c
15.	c
16.	c
17.	a
18.	c
19.	a
20.	a
21.	17.49
22.	2261.96
23.	40
24.	200
25.	111.1

Math

1.	d
2.	a
3.	d
4.	d
5.	c
6.	c
7.	d
8.	a
9.	b
10.	c
11.	b
12.	d
13.	a
14.	d
15.	a
16.	b
17.	c
18.	a
19.	c
20.	a
21.	9
22.	9
23.	9
24.	7
25.	6

Solutions

Physics

1.

Sol^n Heat gain = Heat lost

$C_A(16-12) = C_B(19-16) \Rightarrow \frac{C_A}{C_B} = \frac{3}{4}$

$C_B(23-19) = C_C(28-23) \Rightarrow \frac{C_B}{C_C} = \frac{5}{4}$

$\therefore \frac{C_A}{C_C} = \frac{15}{19}$

If θ is the temperature when A and C are mixed then, $C_A(\theta-12) = C_C(28-\theta)$

$\Rightarrow \frac{C_A}{C_C} = \frac{28-\theta}{\theta-12} \Rightarrow \theta = 20.2°C$ Ans

2. c

3.

Sol^n Linear acc^n for rolling, $a = \frac{g\sin\theta}{1+K^2/R^2}$

$a_{cylinder} = \frac{2}{3}g\sin\theta$

For rotation, torque $f \cdot R = I\alpha$

$= MR^2 \cdot \frac{a}{2}$

$f = \frac{M}{3}g\sin\theta$.

$\mu_s = f/N = \frac{\frac{M}{3}g\sin\theta}{Mg\cos\theta} = \frac{\tan\theta}{3}$

For rolling without slipping of a roller down the inclined plane $\boxed{\tan\theta < 3\mu_s}$

4.

Sol^n $V(x) = \frac{x^4}{4} - \frac{x^2}{2}$ (Pot. Energy)

For Max KE, PE must be min.

$\frac{dV}{dx} = 0$ & $\frac{d^2V}{dx^2} > 0$

$\Rightarrow x^3 - x = 0$

$x(x^2-1) = 0 \Rightarrow x = 0, \pm 1$

$\frac{d^2V}{dx^2} = 3x^2 - 1 > 0 \Rightarrow x \gtrless \pm 1/3$

$V_{min} = \left(\frac{1^4}{4} - \frac{1^2}{2}\right)J = -1/4\ J$

$(KE)_{Max} = TE - (PE)_{Min} = 2 - \left(-\frac{1}{4}\right) = 9/4$

$V_{max}^2 = \frac{9}{2} \Rightarrow V_{max} = 2.121\ m/s$

5.

Sol^n Work by gravity, $W_g = mg = 10\ J$

$W_{net} = \Delta KE = \frac{1}{2} \times 10^{-3} \times 50^2$

$= 1.25\ J$

By Work Energy theorem,

$W_g + W_{res} = W_{net}$

$W_{res} = -8.75\ J$

6.

Sol^n $KE = (KE)_T + (KE)_R$

$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{MR^2}{2}\right)\cdot\omega^2 = \frac{3}{4}Mv^2$

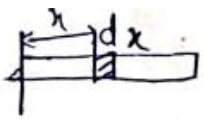
$= \frac{3}{4}(10\times10^{-3}) \times (6\times10^{-2})^2 = 27\ \mu J.$

7.

Sol^n Impulse = Area under F-t graph. It is Max for (III) & (IV)

8.

Sol^n



$x_{cm} = \frac{\int_0^3 (\lambda dx) \times x}{\int_0^3 \lambda\, dx} = \frac{12}{7}\ m$

9.

Sol^n For ce on test charge q_0, $\vec{F_e} = q_0\vec{E_0}$

$W_{i-f} = -\Delta U$

$\Delta U = -W_{i-f} = -q_0E_0(2a-a)$

$= -q_0E_0a$

10. c

11.

Sol² Fraction of supplied energy which increases the internal energy is given by,

$$f = \frac{\Delta U}{(\Delta Q)_p} = \frac{(\Delta Q)_v}{(\Delta Q)_p} = \frac{\mu C_v \Delta T}{\mu C_p \Delta T} = \frac{1}{\gamma}$$

for diatomic gas, $\gamma = \frac{7}{5} \Rightarrow \boxed{f = \frac{5}{7}}$

12.

Sol $\frac{K E_f}{K E_i} = \frac{1}{2} \Rightarrow \frac{v_f}{v_i} = \frac{1}{\sqrt{2}}$

$$\Rightarrow \frac{\sqrt{2gh}}{\sqrt{v_0^2 + 2gh}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow v_0 = 20 \text{ m/s}$$

13.

Sol Induced emf, $\varepsilon = B\frac{dA}{dt} = 3.5 \times 10^{-10}$ V

Induced current, $i = \frac{\varepsilon}{R} = 7.0 \times 10^{-10}$ A.

14.

$$\Delta E = E_2 - E_1 = 13.60\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 10.20 \text{ eV}$$

$$f = \frac{hc}{\Delta E} = 122 \text{ nm}$$

15.

Sol $Y = \overline{\overline{A}.B} = A.B$ = AND gate.

16. b

17.

$$Y = \frac{x}{3z^2} = \frac{M^{-1}L^{-2}T^4A^2}{[MT^{-2}A^{-1}]^2} = [M^{-3}L^{-2}T^8A^4]$$

18.

$$\vec{v} = \vec{u} + \vec{a}t$$
$$= (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j})10$$
$$= 7\hat{i} + 7\hat{j}$$

$$|\vec{v}| = 7\sqrt{2} \text{ unit}$$

19.

Velocity b/w 0 to 2 sec

$$v_1 = \frac{dx}{dt} = \frac{4}{2} = 2 \text{ m/s}$$

velocity at $t = 2$ sec, $v_f = 0$

Impulse = change in momentum $= m(v_f - v_i)$
$= 0.1(0 - 2) = -0.2$ kg m s^{-1}

20.

Sol $u_{\text{balloon}} = 10 + 5 = 15$ m/s upward

after 2 sec, $v = u - gt$
$= 15 - 10 \times 2 = 5$ m/s downward

21.

Sol² $y = A\sin(\omega t + \phi_0)$

$A/2 = A\sin(\omega t + \phi_0) \Rightarrow \phi = \pi/6$ or $5\pi/6$.

phase diff $= \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3} = 120°$

22.

Sol $(C_p)_{mix} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$

$C_{p_1}(He) = \frac{5}{2}R$ and $C_{p_2}(H_2) = \frac{7}{2}R$

$$(C_p)_{mix} = \frac{1 \times \frac{5}{2}R + 1 \times \frac{7}{2}R}{1+1} = 3R = 3 \times 2$$
$$= 6 \text{ Cal/mol°C}$$

∴ Amount of heat needed to raise the temperature 0° to 100 °C $(\Delta Q)_p = nC_p\Delta T = 2 \times 6 \times 100 = 1200$ Cal.

23.

Sol $R_{Max} = \frac{u^2}{g} = 4H \Rightarrow 4H = 400 \Rightarrow \boxed{H = 100 \text{ m}}$

24.

Sol² $\phi = \frac{hc}{\lambda} \Rightarrow \lambda_{max} = \frac{hc}{\phi} = 310$ nm

25.

Sol $V_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow T \propto M$

$$\frac{T_{H_2}}{T_{O_2}} = \frac{M_{H_2}}{M_{O_2}}$$

$$\Rightarrow \frac{T_{H_2}}{(273+47)} = \frac{2}{32} \Rightarrow T_{H_2} = 20\,K$$

Chemistry

1.

Sol ① Given:

Enthalpy of combustion of benzene $= -3268\ kJ\,mol^{-1}$

Enthalpy of combustion of acetylene $= -1300\ kJ\,mol^{-1}$

The change in enthalpy for the reaction $3C_2H_2(g) \rightarrow C_6H_6(l)$

$\Delta H = \Delta H_{reactant} - \Delta H_{product}$

$= 3\times(-1300\ kJ\,mol^{-1}) - (-3268\ kJ\,mol^{-1})$

$= -3900\ kJ\,mol^{-1} + 3268\ kJ\,mol^{-1}$

$= -632\ kJ/mole$

The change in enthalpy for the reaction $3C_2H_2(g) \rightarrow C_6H_6(l)$ is $-632\ kJ/mole$.

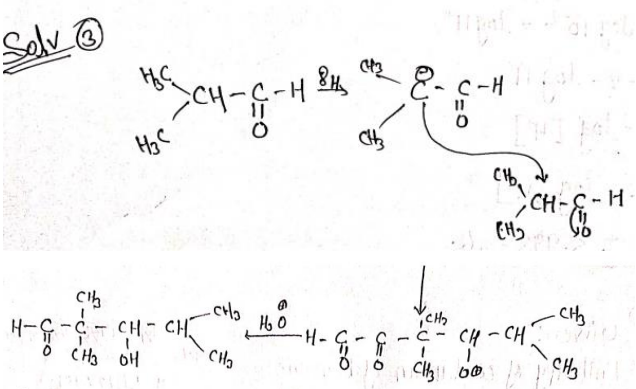
	Type of hybridization	Linearity
$BeCl_2$	Cl–Be–Cl; $H = \frac{1}{2}(2+2-0+0) = 2$; $H = 2 = SP$	
XeF_2	F–Xe–F (3 lone pairs); $H = \frac{1}{2}(8+2-0+0) = 5$; $H = 5 = SP^3d$	linear
CO_2	O=C=O; $H = \frac{1}{2}(4+0-0+0) = 2$; $H = 2 = SP$	Linear

2.

Sol ② Molecules that have identical hybridization would have identical shapes.

3.

Sol ③



Aldol formation takes place.

An aldol condensation is a reaction in which an enol or an enolate ion reacts with a carbonyl compound to form a β-hydroxyaldehyde or β-hydroxyketone.

4.

Sol ④ According to Nernst equation

$$E_{cell} = E^\circ_{cell} - \frac{0.0591}{n}\log\frac{[P]}{[R]}$$

For the given cell

$E_{cell} = 0.576\,V$, $E^\circ_{cell} = 0.34\,V$, $n = 2$

$$0.576 = 0.34 - \frac{0.0591}{2}\log\frac{[H^+]^2}{[Cu^{2+}]}$$

$$0.236 = \frac{0.0591}{2}\times 2\log\frac{[H^+]}{0.01}$$

$$3.993 = [\log H^+ - \log 0.01]$$

$$3.993 = \log 10^{-2} - \log H^+$$

$$3.993 = -2 - \log H^+$$

$$5.993 = -\log[H^+]$$

Also $pH = -\log[H^+]$

$= 5.993 \approx 6.$

5.

⑤

Thin-layer chromatography (TLC) is an adsorption chromatography technique used to separate non-volatile mixtures. Thin-layer chromatography is performed on a sheet of glass, plastic, or aluminium foil, which is coated with a thin layer of adsorbent material, usually silica gel, aluminium oxide (alumina) or cellulose. This layer of adsorbent is known as the stationary phase.

6.

Solv (6) Here,

$n = 2$

$V_1 = 15l$

$V_2 = 50l$

Temperature, $T = 25° = 298K$

Pressure, $P = 1$ atm

Work done $= P(V_2 - V_1)$

$= -1(50-15)$

$= -1 \times 35$

$= -35 l/atm$

As $1 l/atm = 101.3 J$

Therefore, $-35 l atm = -35 \times 101.3$

$= -3545.5 J$

As 1 calorie $= 4.184 J$

So, $-3545.5 J = \frac{-3545.5}{4.184}$ cal

$= -848.2$ cals

7.

Solv (7) N_2O

$2x - 2 = 0$

$\Rightarrow x = +1$

H_3PO_2

$3(1) + x + 2(-2) = 0$

$\Rightarrow x = 9 - 3 = +1$

8.

Solv (8) $N_2O_4 \rightleftharpoons 2NO_2$

At equilibrium $\frac{0.2}{2}$ $\frac{2 \times 10^{-3}}{2}$ (2V)

0.1 1×10^{-3}

$K_c = \frac{(1 \times 10^{-3})^2}{0.1} = 10^{-5}$

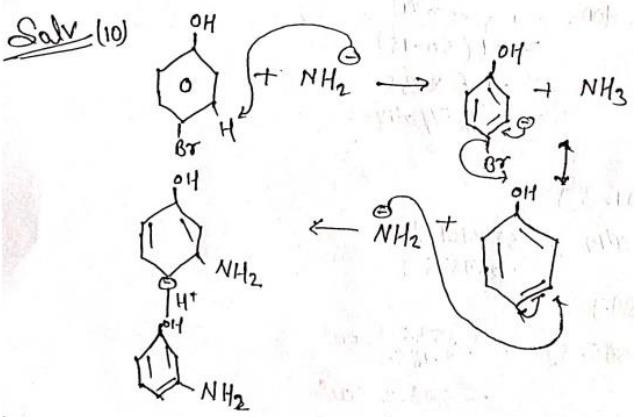
9.

Solv (9) Boric acid H_3BO_3, is monobasic and works as Lewis acid according to the following reaction

$H_3BO_3 + H_2O \rightarrow B(OH)_4^- + H^+$

10.

Solv (10)



11.

IV (11)

$CH_3-\underset{Cl}{CH}-CH_3 \xrightarrow{Alc.KOH/Heat} CH_3CH=CH_2$

2-Chloropropane

$\xrightarrow[Peroxide]{HBr} CH_3-CH_2-CH_2-Br \xrightarrow{aq.KOH}$

$CH_3-CH_2-CH_2-OH$

Propan-1-ol

12.

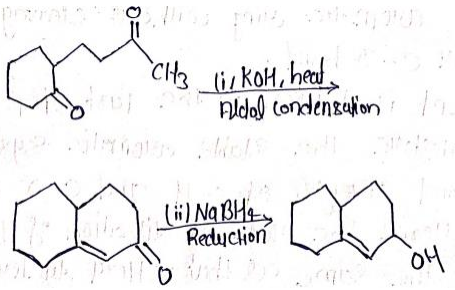
Solv (12) The order of reactivity in S_N1 reaction is mainly dependant on stability of carbocation, formed thus the order of reactivity of the given compounds are as follows.

$CH_3-\underset{CH_3}{\overset{CH_3}{C}}-Cl > CH_3-\underset{Cl}{CH}-CH_3$

$> CH_3-CH_2-Cl > CH_3-Cl$

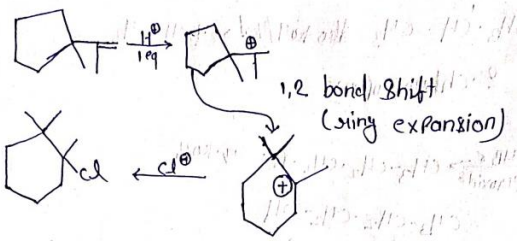
13.

Solv (13)



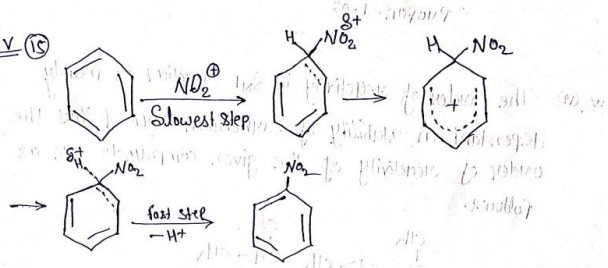
14.

Solv (14)



15.

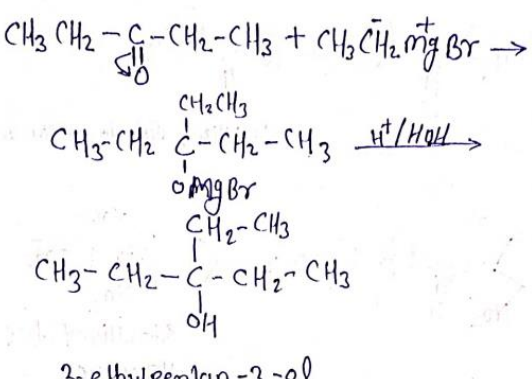
Solv (15)



The rate determining step in electrophilic Substitution reaction, is the bonding of the electrophile to the aromatic ring without cleavage of C-H or C-D bond. This bond is broken in the fast step (second step) that restores the stable aromatic system. Also the bond strength of C-H and C-D bonds are equal. Hence the rate of Nitration of benzene is almost the same as that of Hexa deuterobenzene.

16.

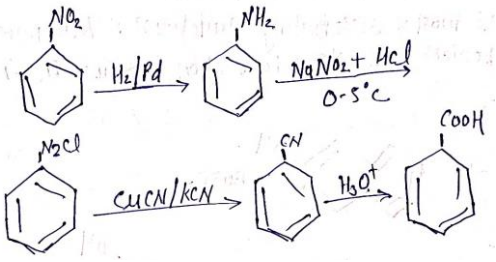
Solv (16)



3-ethylpentan-3-ol

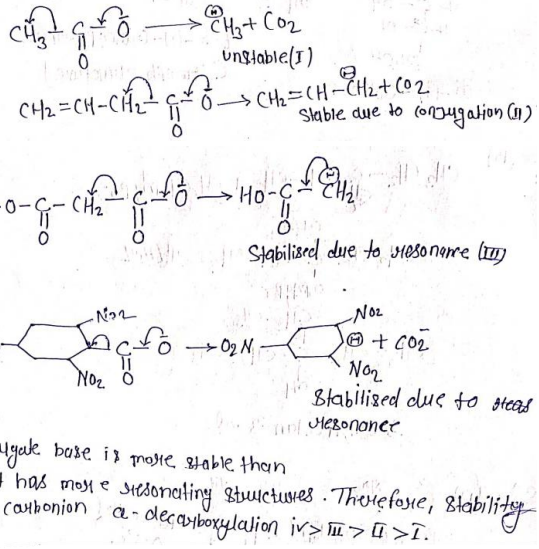
17.

Solv (17)



18.

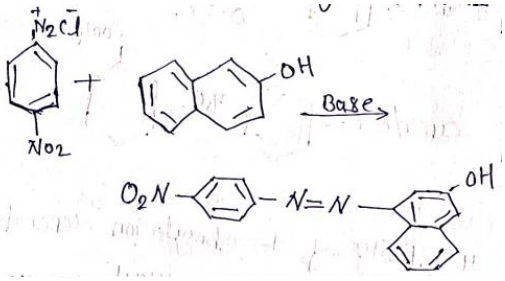
Solv (18) The reactivity of decarboxylation depends upon the stability of the conjugate base. The conjugate bases of the given compounds are as follows:



(IV) conjugate base is more stable than (III) as it has more resonating structures. Therefore, stability of carbonion, α-decarboxylation is IV > III > II > I.

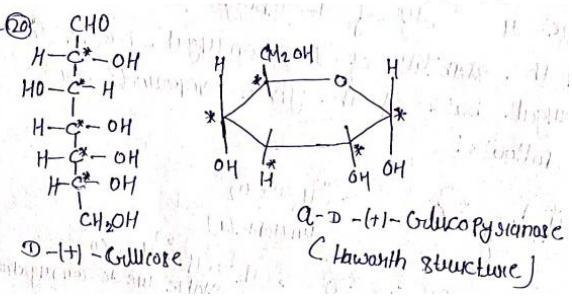
19.

Solv (19)



20.

Solv (20)



21.

(21) Given $k = 0.008\ min^{-1}$

From unit of k, the reaction is a first order reaction.

From: $k = \frac{2.303}{t} \log \frac{V_\infty}{V_\infty - V}$

$\Rightarrow 0.008 = \frac{2.303}{20} \log \frac{V_\infty}{V_\infty - 16}$

$\Rightarrow 0.0695 = \log \frac{V_\infty}{V_\infty - 16}$

$\Rightarrow V_\infty = 17.49\ mL$

22.

Solv (22) According to Gibb's Helmholtz equation,

heart of reaction ΔH, given as

$\Delta H = nF\left[T\left(\frac{\delta E}{\delta T}\right)_p - E\right]$

$T = (273+25)\ K$

$= 298K, n = 2,$

$F = 96500\ C, E = 0.03\ CV$

$\left(\frac{\delta E}{\delta T}\right)_p = 1.4 \times 10^{-4}\ V/K$

$\Delta H = 2 \times 96500\ [298 \times (-1.4 \times 10^{-4})] - 0.03$

$= -1384.2\ J = -13.842\ KJ/mole$

23.

(23)

The given reaction is of the first order with respect to A and of zero order with respect to B.

There fore, the rate of the reaction is given by,

rate $(r) = k[x]^a[y]^b$

k = rate constant

Given that $a = 1; b = 0$

For experiment I:

$r_1 = k[0.1]^a[0.1]^b = 2 \times 10^{-3}$

$k[0.1]^1[0.1]^0 = 2 \times 10^{-3}$

For experiment II:

$r_{II} = k[L]^1[0.2]^0 = 4 \times 10^{-3}$

equation (ii) ÷ equation (i)

$\frac{L}{0.1} = \frac{4 \times 10^{-3}}{2 \times 10^{-3}}$

$L = 0.2$

For experiment III:

$r_{III} = k[0.4]^1[0.4]^0 = M \times 10^{-3}$

For experiment IV:—

$r_{IV} = k[0.1]^1[0.2]^0 = 2 \times 10^{-3}$

Divide equation (iii) by equation (iv):—

$\frac{0.4}{0.1} = \frac{M \times 10^{-3}}{2 \times 10^{-3}} \Rightarrow M = 8$

$\frac{M}{L} = \frac{8}{0.2} = 40$ } Ratio of M and L = 40

24.

(24) $C_{(s)} + O_{2(g)} \longrightarrow CO_{2(g)};\ \Delta H = -K]/mol$

From

$Q = C\Delta T$

$Q = 20\ KJ \times 2.$

40KJ of heat is released from 2.4 gm of C-atom

For 1 mole of C-atom

$Q = \frac{40}{2.4} \times 12$

$Q = \frac{40}{2.4} \times 12 = 200KJ/mol$

From $\Delta H = \Delta E + \Delta n_g RT$

$\Delta n_g = 0,\ \Delta H = \Delta E$

$Q = \Delta H = \Delta E$

$\Delta H = 200$

25.

Solv (25) Assume the vapour pressure of water = 100

∴ Vapour pressure of urea solution = 75

Weight of urea = w_1

Molecular weight of urea = Mw_1

Weight of water = w_2

Molecular weight of water = Mw_2

By Raoult's law

$$= \frac{\frac{w_1}{Mw_1}}{\frac{w_2}{Mw_2} \frac{w_1}{Mw_1}}$$

So, $\frac{100-75}{100} = \frac{60}{\frac{100}{18}+\frac{w_1}{60}}$

$\Rightarrow$ weight of urea $w_1 = 111.1\,g$

Mathematics

① $f(x) = |x-1| \cdot ([x] - [-x])$

$$f(x) = \begin{cases} |x-1|(2[x]) & x \in z \\ |x-1|(2[x]+1) & x \notin z \end{cases}$$

at $x=1$ Continuity

$f(1) = 0 \quad f(1^-) = 0 \quad f(1^+) = 0$ } → continuous

Differentiability

$$\lim_{h\to 0} \frac{f(1-h)-f(1)}{-h} \Rightarrow \lim_{h\to 0} \frac{h-0}{-h} = -1 \quad (LHD)$$

$$\lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{h(3)-0}{h} = 3 \quad (RHD)$$

LHD ≠ RHD Not differentiable

② A fair dice is thrown three times

If product of outcomes is a prime number, then one number must be prime number and others must be 1

so we have the following possible outcomes

1, 1, 2
1, 1, 3
1, 1, 5

Probability $= \frac{3+3+3}{216} = \frac{1}{24}$

③ Let $S(h,k)$

$SQ^2 = (h+1)^2 + k^2 \quad SR^2 = (h-2)^2 + k^2 \quad SP^2 = (h-1)^2 + k^2$

$SQ^2 + SR^2 = 2SP^2$

$h^2+1+2h+k^2 + h^2+4-4h+k^2 = 2(h^2+1-2h+k^2)$

$-2h+5 = -4h+2$

$2h = -3$

$h = -\frac{3}{2} \Rightarrow x = -\frac{3}{2}$

Ans: (d)

④ $(xy^3 + x^2y^7)\frac{dy}{dx} = 1$

$\frac{dx}{dy} = xy^3 + x^2y^7$

Divide by x^2

$\frac{1}{x^2}\frac{dx}{dy} - \frac{1}{x}y^3 = y^7$

Let $-\frac{1}{x} = u$

$\frac{1}{x^2}\frac{dx}{dy} = \frac{du}{dy}$

$\frac{du}{dy} + uy^3 = y^7$

$IF = e^{\int y^3 dy} \Rightarrow e^{y^4/4}$

$\Rightarrow (e^{y^4/4})u = \int y^7 e^{y^4/4} dy$

Let $\frac{y^4}{4} = m$

$y^3 dy = dm$

$(e^{y^4/4})(-\frac{1}{x}) = \int (4m) e^m dm$

$\Rightarrow 4\int m e^m dm \Rightarrow 4[me^m - e^m] + C$

(I) (II)

$(e^{y^4/4})(-\frac{1}{x}) = 4(\frac{y^4}{4})e^{y^4/4} - 4e^{y^4/4} + C$

$-\frac{e^{y^4/4}}{x} = y^4 e^{y^4/4} - 4e^{y^4/4} + C$

Put $(1, \frac{1}{4})$

$-4e^{1/4} = e^{1/4} - 4e^{1/4} + C \Rightarrow C = -e^{1/4}$

Solution

$-\frac{e^{y^4/4}}{x} = y^4 e^{y^4/4} - 4e^{y^4/4} - e^{1/4}$, when $y=1, x=\frac{1}{4}$

Ans:

$\frac{dy}{dx} = \frac{1}{xy^3+x^2y^7}$

$\Rightarrow \frac{1}{-\frac{1}{4}-\frac{1}{16}} \Rightarrow -\left(\frac{16}{5}\right)$

⑤ $f(x) + 2f\left(\frac{2x+1}{x-2}\right) = 3x \quad x \neq 2$

Put $x=3$: $f(3) + 2f(7) = 9$

Put $x=7$: $f(7) + 2f(3) = 21$

Solve both :-

$f(3) + 2f(7) = 9$

$4f(3) + 2f(7) = 42$

$-3f(3) = -33$

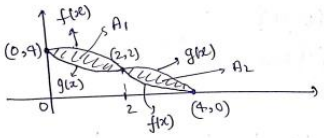
$f(3) = 11$

$f(7) + 22 = 21$

$f(7) = -1$

$\frac{f(3)}{f(7)} = \frac{11}{-1} = -11$ Ans

⑥



$\Rightarrow \int_0^4 (-f(x) - g(x)) = 10$

$\Rightarrow \int_0^2 \{f(x) - g(x)\} + \int_2^4 \{f(x) - g(x)\} = 10$

$\Rightarrow \int_0^2 \{f(x) - g(x)\} = 15$ Ans

⑦ Let $A=\begin{bmatrix} m & n \\ p & q \end{bmatrix}$ $\quad adj(A)=\begin{bmatrix} q & -n \\ -p & m \end{bmatrix}$

$|A| = mq - np$ $\quad (|A| = d)$

$d\,adj(A) = \begin{bmatrix} qd & -nd \\ -pd & md \end{bmatrix}$ $\quad A + d\,adj(A) = \begin{bmatrix} m & n \\ p & q \end{bmatrix} + \begin{bmatrix} qd & -nd \\ -pd & md \end{bmatrix}$

$\Rightarrow \begin{bmatrix} m+qd & n(1-d) \\ p(1-d) & q+md \end{bmatrix}$

$|A + d\,adj\,A| = 0$

$\begin{vmatrix} m+qd & n(1-d) \\ p(1-d) & q+md \end{vmatrix} = 0$

$(m+qd)(q+md) - pn(1-d)^2 = 0$

$mq + m^2d + q^2d + mqd^2 - np - npd^2 + 2npd = 0$ $\quad$ Put $np = d - mq$

$(mq - np) + (mq - np)d^2 + m^2d + q^2d + 2mqd - 2d^2 = 0$

$(d + d^3 - 2d^2) + d(m^2 + q^2 + 2mq) = 0$

$d(1 + d^2 - 2d) + d(m+q)^2 = 0$

$d[(d-1)^2 + (m+q)^2] = 0$ $\quad (d \neq 0)$

$(d = 1)$ $\quad (m = -q)$

$|A - |A|\,adj(A)| = |A - adj(A)|$

$\Rightarrow \left| \begin{pmatrix} m & n \\ p & q \end{pmatrix} - \begin{pmatrix} q & -n \\ -p & m \end{pmatrix} \right|$

$\Rightarrow \begin{vmatrix} m-q & 2n \\ 2p & q-m \end{vmatrix} \Rightarrow -4m^2 - 4pn$

$\Rightarrow -4(m^2 + pn)$

$\Rightarrow -4(pn - mq) = 4$ Ans

⑧ case① all three digits are alike

$111, 333, 555, 888 \longrightarrow 4$

case② 2 digits are alike

$558 \longrightarrow \frac{3!}{2!} = 3$, $885 \longrightarrow \frac{3!}{2!} = 3$

case③ all three digits are different

$1,3,5 \longrightarrow 6$ $\quad 1,3,8 \longrightarrow 6$

Total numbers $= 22$ Ans

⑨ $\int \frac{x + x^{2/3} + 2x^{1/6}}{x(1 + x^{1/3})}\,dx$

Substitute $x = u^6$ $\quad dx = 6u^5\,du$

$\int \frac{(u^6 + u^4 + 2u)6u^5}{u^6(1+u^2)}\,du \Rightarrow 6\int \frac{(u^5 + u^3 + 2)u^6}{u^6(1+u^2)}\,du$

$\Rightarrow 6\int \frac{u^5 + u^3 + 2}{1+u^2}\,du$

$\Rightarrow 6\left(\int \frac{u^3(1+u^2)}{1+u^2} + \int \frac{2}{1+u^2} \right)$

$\Rightarrow 6\frac{u^4}{4} + 12\tan^{-1}u + C$

$\Rightarrow \frac{3}{2}x^{2/3} + 12\tan^{-1}(x^{1/6}) + C$ Ans

⑩ $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}} \right)^{10}$

$\left(\frac{(x^{1/3})^3 + 1^3}{x^{2/3} - x^{1/3} + 1} - \frac{(x^{1/2})^2 - 1^2}{x^{1/2}(x^{1/2} - 1)} \right)^{10}$

$\left(\frac{(x^{1/3}+1)(x^{2/3} - x^{1/3} + 1)}{x^{2/3} - x^{1/3} + 1} - \frac{(x^{1/2} - 1)(x^{1/2} + 1)}{x^{1/2}(x^{1/2} - 1)} \right)^{10}$

$\left(x^{1/3} + 1 - 1 - x^{-1/2} \right)^{10} \Rightarrow \left(x^{1/3} - x^{-1/2} \right)^{10}$

$\Rightarrow {}^{10}C_r \left(x^{1/3}\right)^r \left(-x^{-1/2}\right)^{10-r}$

$\Rightarrow {}^{10}C_r\, x^{r/3} (-1)^{10-r}\, x^{-\left(\frac{10-r}{2}\right)}$

$\Rightarrow (-1)^{10-r}\, {}^{10}C_r\, x^{\left(\frac{r}{3} + \frac{10-r}{2}\right)}$

For Independent of x

$\frac{r}{3} + \frac{10}{2} + \frac{r}{2} = 0$

$-5 + \frac{2r+3r}{6} = 0$ $\quad \frac{5r}{6} = 5$

$5r = 30$ $\quad (r = 6)$

$\Rightarrow$ Independent term $\Rightarrow {}^{10}C_6 \Rightarrow \frac{10!}{6!\,4!}$

$\Rightarrow \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \Rightarrow (210)$ Ans

⑪ $|\vec{OP}|^2 = \left(t + \frac{1}{t}\right)^2 |\vec{a}|^2 + \left(t - \frac{1}{t}\right)^2 |\vec{b}|^2 + 2\left(t^2 - \frac{1}{t^2}\right)|\vec{a}||\vec{b}| \cos\frac{2\pi}{3}$

$\Rightarrow 9\left(t + \frac{1}{t}\right)^2 + 4\left(t - \frac{1}{t}\right)^2 + 2\left(t^2 - \frac{1}{t^2}\right)3 \cdot 2 \cdot \left(-\frac{1}{2}\right)$

$\Rightarrow 7t^2 + \frac{19}{t^2} + 10$

$|\vec{OP}|^2 \geq 2\sqrt{7t^2 \cdot \frac{19}{t^2}} + 10$ $\quad (AM \geq GM)$

$|\vec{OP}|^2 \geq 2\sqrt{133} + 10$

$|\vec{OP}| \geq \sqrt{10 + 2\sqrt{133}}$ Ans

⑫ $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} = \sum \frac{1}{r}$

$S_n = \frac{1^2}{1^3} + \frac{1^2 + 2^2}{1^3 + 2^3} + \frac{1^2 + 2^2 + 3^2}{1^3 + 2^3 + 3^3} + \cdots$

$T_r = \frac{1^2 + 2^2 + 3^2 + \cdots + r^2}{1^3 + 2^3 + 3^3 + \cdots + r^3} = \frac{\frac{r(r+1)(2r+1)}{6}}{\frac{r^2(r+1)^2}{4}}$

$T_r = \frac{2}{3}\left(\frac{2r+1}{r(r+1)}\right)$

$S_n = \sum T_r = \sum \frac{2}{3}\left[\frac{r + r + 1}{r(r+1)}\right] = \sum \frac{2}{3}\left[\frac{1}{r} + \frac{1}{r+1}\right]$

$S_n = \frac{2}{3}\left[H_n + H_{n-1} + \frac{1}{n+1}\right] \Rightarrow \left[\frac{4}{3}H_n - \frac{2}{3}\left(\frac{n}{n+1}\right)\right]$ Ans

$\sum \frac{1}{r+1} = \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \frac{1}{n+1} \Rightarrow H_{n-1} + \frac{1}{n+1}$

(13) $w_i = lx_i + k$

$\sum w_i = l\sum x_i + nk$

$\frac{1}{n}\sum w_i = l\left(\frac{1}{n}\sum x_i\right) + k$

$55 = 48l + k \quad -(i)$

$w_i = lx_i + k$

$Var(w) = l^2 Var(x)$

$\sigma_w = |l|\sigma_x$

$15 = 12|l| \Rightarrow |l| = \frac{5}{4} = 1.25$

~~$l = 1.25$~~ ~~$k = -5$~~ ~~$k = 115$~~

$l = 1.25 \mid k = -5$

$l = -1.25 \mid k = 115$

Ans: 9

(14) $\alpha^2 - \alpha + 2d = 0 \quad -(i)$

$3\alpha^2 - 10\alpha + 27d = 0 \quad -(ii)$

(ii) -3(i)

$3\alpha^2 - 10\alpha + 27d = 0$

$3\alpha^2 - 3\alpha + 6d = 0$

$-7\alpha + 21d = 0$

$\alpha = 3d$

Put in (i)

$9d^2 - 3d + 2d = 0$

$9d^2 - d = 0$

$d(9d-1) = 0$

$d \neq 0 \quad d = \frac{1}{9}$

$\alpha = \frac{1}{3}$

~~$\alpha\beta$~~

$\alpha + \beta = 1$

$\frac{1}{3} + \beta = 1$

$\beta = \frac{2}{3}$

$\alpha + \gamma = \frac{10}{3}$

$\frac{1}{3} + \gamma = \frac{10}{3}$

$\gamma = \frac{10}{3} - \frac{1}{3} = 3$

$\gamma = 3$

$\frac{\beta\gamma}{d} = \frac{2}{3} \times \frac{3}{1} \times 9 \Rightarrow 18$ Ans

(15) $\left(\frac{2z - 3i}{4z + 2i}\right) \rightarrow$ Real $\Rightarrow Im\left(\frac{2z-3i}{4z+2i}\right) = 0$

$\frac{2(x+iy) - 3i}{4(x+iy) + 2i} \Rightarrow \frac{2x + i(2y-3)}{4x + i(4y+2)} \times \frac{4x - i(4y+2)}{4x - i(4y+2)}$

$\Rightarrow$

$Im = \frac{4x(2y-3) - 2x(4y+2)}{16x^2 + (4y+2)^2} \Rightarrow \frac{8xy - 12x - 8xy - 4x}{16x^2 + 16y^2 + 16y + 4}$

$Im = \frac{-16x}{16x^2 + 16y^2 + 16y + 4} = 0$

$x = 0 \qquad 16x^2 + 16y^2 + 16y + 4 \neq 0$

$x^2 + y^2 + y \neq -\frac{1}{4}$

If $x = 0$ then

$y^2 + y \neq -\frac{1}{4}$

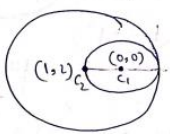
$4y^2 + 4y + 1 \neq 0$

$(2y+1)^2 \neq 0 \Rightarrow y \neq -\frac{1}{2}$

(16) $C_1 : x^2 + y^2 = 5 \qquad C_2 : x^2 + y^2 - 2x - 4y + 15 = 0$

$C_1(0,0) \quad r_1 = \sqrt{5} \qquad C_2(1,2) \quad r_2 = \sqrt{1+4+15} \Rightarrow 2\sqrt{5}$

$C_1C_2 = \sqrt{1+4} = \sqrt{5}$



(17) $\lim_{x\to 1} \sec^{-1}\left(\frac{d^2}{\ln x} - \frac{d^2}{x-1}\right) = \lim_{x\to 1} \sec^{-1}\left[d^2\left(\frac{1}{\ln x} - \frac{1}{x-1}\right)\right]$

$\Rightarrow \lim_{x\to 1} \sec^{-1}\left(d^2\left(\frac{x-1-\ln x}{(\ln x)(x-1)}\right)\right)$

Use L-Hopital Rule to find

$\Rightarrow \lim_{x\to 1} \frac{x-1-\ln x}{\ln x(x-1)} = \frac{1}{2}$

$\Rightarrow \lim_{x\to 1} \sec^{-1}\left(d^2\left(\frac{x-1-\ln x}{\ln x(x-1)}\right)\right) = \sec^{-1}\left(\frac{d^2}{2}\right)$

$\frac{d^2}{2}$ should not be in $(-1,1)$

$-1 \leq \frac{d^2}{2} \leq 1 \Rightarrow d^2 \leq 2$

$d \notin (-\sqrt{2}, \sqrt{2})$

$d \neq -1, 0, 1$

(18) $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = d \qquad (2d+1, 3d+2, 4d+3)$

$\frac{x-9}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu \qquad (5\mu+9, 2\mu+1, \mu)$

$2d+1 = 5\mu + 9$

$3d+2 = 2\mu + 1$

$4d+3 = \mu$

$\rightarrow$ solve

$2d+1 = 5(4d+3) + 9$

$2d+1 = 20d + 15 + 9$

$-18 = 18d \Rightarrow d = -1$

$\mu = -1$

POI $= (-1, -1, -1)$

Line through $(-1,-1,-1), (2,1,2)$

DR of $\langle 3, 2, -1 \rangle$

$\frac{x+1}{3} = \frac{y+1}{2} = \frac{z+1}{-1} = t \Rightarrow (3t-1, 2t-1, -t-1)$

Point $(-4, -3, 0)$

on xy plane $-t - 1 = 0 \Rightarrow t = -1$

(19) $f(x) \cdot f'(x) \geq x\sqrt{1 - (f(x))^4}$

$\frac{f(x) \cdot f'(x)}{\sqrt{1-(f(x))^4}} - x \geq 0$ multiply by 2

$\frac{2f(x) \cdot f'(x)}{\sqrt{1-(f(x))^4}} - 2x \geq 0$

$\frac{d}{dx}\left(\sin^{-1}(f(x)^2) - x^2\right) \geq 0$

Then $g(x) = \sin^{-1}(f(x)^2) - x^2$ is a non-decreasing fn

$\lim_{x\to x_1^+} g(x) \leq \lim_{x\to x_2^-} g(x)$

$\frac{\pi}{2} - x_1^2 \leq \frac{\pi}{6} - x_2^2$

$x_1 - x_2^2 \geq \frac{\pi}{3}$ Ans

(20) $A = \begin{pmatrix} 1 & 2 & 2^2 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{2^2} & \frac{1}{2} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 4 \\ 1/2 & 1 & 2 \\ 1/4 & 1/2 & 1 \end{pmatrix}$

$A^2 = \begin{pmatrix} 1 & 2 & 2^2 \\ 1/2 & 1 & 2 \\ 1/2^2 & 1/2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 2^2 \\ 1/2 & 1 & 2 \\ 1/2^2 & 1/2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 6 & 12 \\ 3/2 & 3 & 6 \\ 3/4 & 3/2 & 3 \end{pmatrix}$

$A^2 = 3A$, ~~$A^3 = 3A^2$~~ $A^3 = A \cdot A^2 \Rightarrow 3A^2 = 3^2 A$.

$A^2 + A^3 + \cdots + A^{10} = 3A + 3^2A + \cdots + 3^9 A \Rightarrow \frac{3(3^9 - 1)}{3-1} A$.

$\Rightarrow \left(\frac{3^{10} - 3}{2}\right) A$

① $\lim_{x\to 0}\left(\frac{1}{\ln(1+x)}-\frac{1}{\ln(x+\sqrt{1+x^2})}\right)$ $(\infty-\infty)$

$\lim_{x\to 0}\frac{\ln(x+\sqrt{1+x^2})-\ln(1+x)}{\ln(1+x)\ln(x+\sqrt{1+x^2})}$ $\left(\frac{0}{0}\right)$

Use L-Hopital

$\lim_{x\to 0}\frac{\left(\frac{1}{x+\sqrt{1+x^2}}\right)\left(1+\frac{x}{\sqrt{1+x^2}}\right)-\frac{1}{1+x}}{\frac{1}{1+x}\ln(x+\sqrt{1+x^2})+\frac{\ln(1+x)}{(x+\sqrt{1+x^2})}\left(1+\frac{x}{\sqrt{1+x^2}}\right)}$

$\lim_{x\to 0}\frac{\frac{1}{\sqrt{1+x^2}}-\frac{1}{1+x}}{\frac{\ln(x+\sqrt{1+x^2})}{1+x}+\frac{\ln(1+x)}{\sqrt{1+x^2}}}$

$\lim_{x\to 0}\frac{1+x-\sqrt{1+x^2}}{\sqrt{1+x^2}\ln(x+\sqrt{1+x^2})+(1+x)\ln(1+x)}$

$\lim_{x\to 0}\frac{1-\frac{x}{\sqrt{1+x^2}}}{\frac{x}{\sqrt{1+x^2}}\ln(x+\sqrt{1+x^2})+\frac{\sqrt{1+x^2}}{x+\sqrt{1+x^2}}\left(1+\frac{x}{\sqrt{1+x^2}}\right)+1+\ln(1+x)}$

$\frac{1-0}{0+1+1+0}\Rightarrow\boxed{\frac{1}{2}}$ $\quad 1\&2=\boxed{9}$ Ans

② $\int_0^x(f'(t)-\sin 2t)\,dt=\int_x^0(f(t)\tan t)\,dt$

$f'(x)-\sin 2x=-f(x)\tan x$

$f'(x)+f(x)\tan x=\sin 2x$

$\frac{dy}{dx}+y\tan x=\sin 2x$

$IF=e^{\int\tan x\,dx}\Rightarrow e^{\ln\sec x}=\sec x$

$y(\sec x)=\int(\sin 2x)\sec x\,dx$

$\Rightarrow\int\frac{2\sin x\cos x}{\cos x}\,dx\Rightarrow 2(-\cos x)+C$

$\frac{f(x)}{\cos x}=-2\cos x+C$

$\frac{1}{1}=-2+C\Rightarrow\boxed{C=3}$

$f(x)=-2\cos^2x+3\cos x\Rightarrow -2\left(\cos^2x-\frac{3}{2}\cos x\right)$

$\Rightarrow -2\left(\cos^2x-\frac{\cos x}{1/2}+\frac{9}{16}-\frac{9}{16}\right)$

$\Rightarrow\frac{9}{8}-2\left(\cos x-\frac{3}{4}\right)^2$

$\boxed{\max(f(x))=\frac{9}{8}}$

③ $L_1:\ \frac{x-7}{-3}=\frac{y-6}{2}=\frac{z-2}{4}$ $\quad C(-3\lambda+7,2\lambda+6,4\lambda+2)$

$L_2:\ \frac{x-5}{2}=\frac{y-3}{1}=\frac{z-4}{3}$ $\quad D(2\mu+5,\mu+3,3\mu+4)$

$L_3=\frac{x+3\lambda-7}{2}=\frac{y-2\lambda-6}{-2}=\frac{z-4\lambda-2}{-1}$

Put D

$\frac{2\mu+5+3\lambda-7}{2}=\frac{\mu+3-2\lambda-6}{-2}=\frac{3\mu+4-4\lambda-2}{-1}$

$-\mu-3+2\lambda+6=-6\mu-8+8\lambda+8$

$5\mu-6\lambda=-7\ -(ii)$

$-4\mu-10-6\lambda+14=2\mu+6-4\lambda-12$

$6\mu+2\lambda=10\ -(i)$

$3(i)+(ii)$

$18\mu+6\lambda=30$

$5\mu-6\lambda=-7$

$23\mu=23$

$\boxed{\mu=1}$

Put (ii)

$5-6\lambda=-7$

$6\lambda=12$

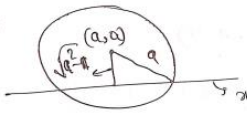
$\boxed{\lambda=2}$

$C(1,10,10)\quad D(7,4,7)$

$|\vec{CD}|=\sqrt{36+36+9}\Rightarrow\sqrt{81}=9$ Ans

④ Let center of circle (a,a), so radius $=a$

eqⁿ of line $x+y=2$



$\sqrt{a^2-1}=\left|\frac{a+a-2}{\sqrt{2}}\right|\Rightarrow a^2-1=2(a^2+1-2a)$

$\Rightarrow a^2-4a+3=0$

$r_1+r_2=4,\ r_1r_2=3$

Roots of this eqⁿ is both circles radius

$r_1^2+r_2^2-r_1r_2=(r_1+r_2)^2-3r_1r_2$

$\Rightarrow 16-9=7$ Ans

⑤ Eqⁿ of normal $y=-tx+2at+at^3$ (Parabola $x=4y^2$)

$y=-tx+\frac{2}{16}t+\frac{1}{16}t^3$

it passes through $(0,33)$ (Shortest distance is always along normal)

$33=\frac{t}{8}+\frac{t^3}{16}$

$t^3+2t-528=0$

$(t-8)(t^2+8t+66)=0$

$\boxed{t=8}$

Point $P(at^2,2at)=(4,1)$

Parabola :-

$y^2=4(x+y)$

$y^2-4y=4x$

$(y-2)^2=4(x+1)$

Eqⁿ of directrix $\Rightarrow x+1=-1$

$\boxed{x=-2}$

Distance $=6$